Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

Lecture: 06

Fourier Transform

* We have studied Fourier series and we knew it deals with periodic signals (power signals).
* Fourier transform Deals with aperiodic signals.
* If the signal is aperiodic, we assume it repeats itself at ∞, then

$$f(t) \xleftrightarrow{F.T.} F(\omega)$$

where $F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$ [Foreword]

and

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega \quad \text{[inverse]}$$

Another form of Fourier transform is

$$F(f) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi f t}\, dt \qquad [\text{Foreword}]$$

and

$$f(t) = \int_{-\infty}^{\infty} F(f)\, e^{j2\pi f t}\, df \qquad [\text{inverse}]$$

* in this course, mostly, we will use the second form.

* Fourier transform plays an important and significant role in communication systems.

* By making use of Fourier transform properties, differenet values can be obtained, such as simplifying problems and easy designes.

* However, Fourier transform, transforms time-domain Signals to Frequency-domain signals.

EX.1 Find the Fourier transform of e^{-at} $t \geq 0$.

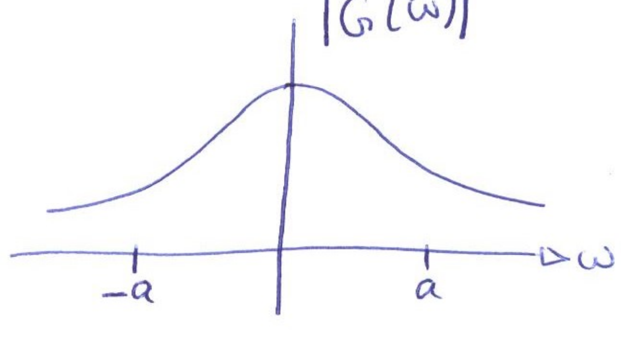
solution $g(t) = e^{-at}$ $t \geq 0$

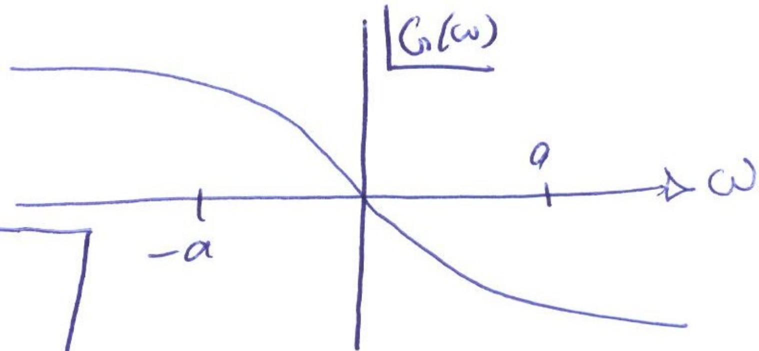
$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt = \int_{0}^{\infty} e^{-at}\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j2\pi f)t}\, dt = \frac{-1}{a+j2\pi f}\, e^{-(a+j2\pi f)t}\Big|_{0}^{\infty}$$

$$= \frac{-1}{a+j2\pi f}\left[0 - 1\right]$$

$\therefore$ $\boxed{G(f) = \dfrac{1}{a+j2\pi f}}$





* Generally:

$$\boxed{e^{\mp at} \xleftrightarrow{\text{F.T.}} \frac{1}{a \pm j\omega}}$$

EX. 2 Find Fourier transform of $g(t) = \delta(t-5)$ and sketch the magnitude and phase spectrums.

Solution

$$G(f) = \int_{-\infty}^{\infty} \delta(t-5)\, e^{-j2\pi f t}\, dt$$

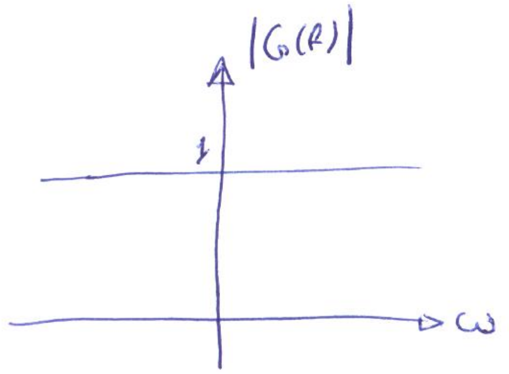
* we know that $\int_{-\infty}^{\infty} \delta(t-t_0) f(t)\, dt = f(t_0)$, then

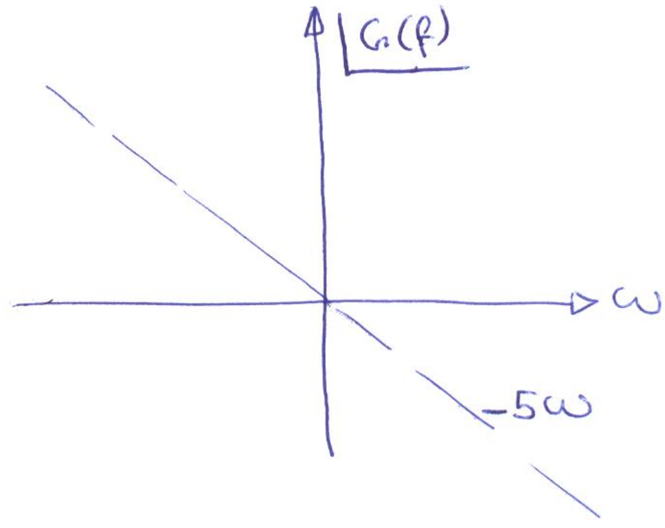
$$G(f) = \int_{-\infty}^{\infty} \delta(t-5)\, e^{-j2\pi f t}\, dt = e^{-j2\pi f 5}$$

Thus $\boxed{G(f) = e^{-j\omega 5}}$ $\Rightarrow e^{-j5\omega} = \cos(5\omega) - j\sin(5\omega)$

$$|G(f)| = |e^{-j5\omega}| = 1 \quad \text{for all } \omega$$

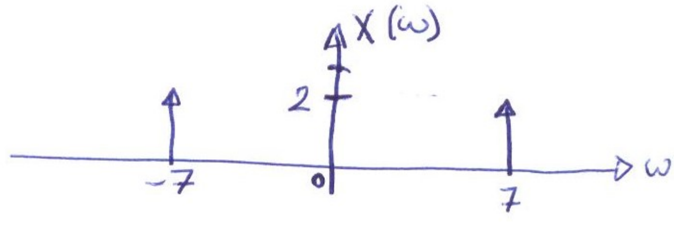
$$\angle G(f) = \tan^{-1}\left(\frac{-\sin(5\omega)}{\cos(5\omega)}\right) = -5\omega$$





EX. 3 Find the inverse Fourier transform of the signal shown below.

Solution



From the Figure:

$$X(\omega) = 2\,\delta(\omega+7) + 2\,\delta(\omega-7)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega = \frac{2}{2\pi}\int_{-\infty}^{\infty} \delta(\omega+7)\, e^{j\omega t}\, d\omega + \frac{2}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-7)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{\pi} e^{-j7t} + \frac{1}{\pi} e^{j7t} = \frac{2}{\pi}\cos(7t).$$

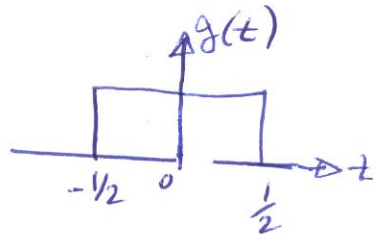
EX. 4 Find $F\{\delta(t)\}$.

Solution $\quad g(t) = \delta(t)$

$$G(f) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi f t}\, dt = 1$$

Ex. 4 Find the Fourier transform of $g(t) = 2\,\mathrm{rect}(t-2)$.

Solution $g(t) = 2\,\mathrm{rect}(t-2)$



$$G(f) = 2\int_{-1/2}^{1/2} \mathrm{rect}(t-2)\, e^{-j2\pi f t}\, dt$$

Let $t-2 = \tau \rightarrow dt = d\tau$ and $t = \tau + 2$

$$G(f) = 2\int_{-1/2}^{1/2} \mathrm{rect}(\tau)\, e^{-j2\pi f(\tau+2)}\, d\tau = \int_{-1/2}^{1/2} e^{-j2\pi f\tau}\, e^{-j2\pi f 2}\, d\tau$$

$$= 2e^{-j2\pi f 2}\int_{-\infty}^{\infty} e^{-j2\pi f\tau}\, d\tau = 2e^{-j2\pi f 2}\left[\int_{-\infty}^{\infty}\Big(\underbrace{\cos(2\pi f\tau)}_{\text{even}} - j\,\underbrace{\overbrace{\sin(2\pi f\tau)}^{\text{zero}}}_{\text{odd}}\Big)\, d\tau\right]$$

$$= 2e^{-j2\pi f 2}\, 2\int_{0}^{1/2} \cos(2\pi f\tau)\, d\tau = \frac{2*2\, e^{-j2\pi f 2}}{2\pi f}\sin(2\pi f\tau)\Big|_{0}^{1/2}$$

$$= \frac{4e^{-j2\pi f 2}}{2\pi f}\sin\left(\frac{2\pi f}{2}\right) = \frac{2e^{-j2\pi f 2}}{\pi f}\sin(\pi f)$$

$$= 2e^{-j2\pi f 2}\,\frac{\sin(\pi f)}{\pi f} = 2e^{-j2\omega}\,\mathrm{sinc}(f)$$

EX.5 Find the Fourier transform of $\cos(2\pi f_0 t)$.

solution

$$g(t) = \cos(2\pi f_0 t) = \cos(\omega_0 t)$$

$$G(f) = \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-j\omega t}\, dt$$

$\Rightarrow$ Since $e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)$

$$G(f) = \int_{-\infty}^{\infty} \cos(\omega_0 t)\cos(\omega t)\, dt - j \underbrace{\int_{-\infty}^{\infty} \cos(\omega_0 t)\sin(\omega t)\, dt}_{\text{zero}}$$

$$= \int_{-\infty}^{\infty} \cos(2\pi f_0 t)\cos(2\pi f t)\, dt$$

* if $f = f_0 \Rightarrow \cos(2\pi f)\cos(2\pi f) = \cos^2(2\pi f)$
* if $f = -f_0 \Rightarrow \cos(2\pi f)\cos(-2\pi f) = \cos^2(2\pi f)$

Hence: only when $f_0 = \mp f$ the integration is non-zero and equals to $\frac{1}{2}$, then

$$G(f) = \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$$